The Mission Planning of Multi-wave Missile Launching Based on 0-1 Integer Programming

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Abstract: In this paper, an optimization problem based on multi-wave missile launching mission planning and maneuvering route was studied. Firstly, a 0-1 integer programming model was established by using the global optimization, and then the exposure time was defined as three parts: Road maneuvering time, node waiting time and waiting time for shooting at the same time. Because the road maneuvering time was the main time of them, the shortest path based on it was solved by Dijkstra algorithm. Finally, the results of global optimization were obtained by adjusting the latter two parts of time. The study of this problem provides a method for the task assignment problem with multi-waves, and can be extended to n waves easily.

1. Introduction

In order to improve the survival probability and damage effect of the target, the missile often carries out multi-wave missile strike and maneuvers based on the route network. With the rapid development of battlefield reconnaissance and surveillance, missiles are easy to be exposed to the enemy's reconnaissance range in the process of maneuver. Therefore, reasonable planning of mobile routes to reduce exposure time has become a key issue in the study of missile launch. At present, one-wave mission planning can be simplified to the shortest path problem, which is more mature, but the multi-wave mission planning problem is more complex, and there is no good method at present.

A certain unit has 12 launchers, which are deployed in two standby areas (D_1, D_2) on average. There are 30 launch sites (F_1-F_{30}), 5 loading areas (Z_1-Z_5) and 38 road nodes (J_1-J_{38}) in the operation area. The names and coordinates of the standby areas, the loading areas, the launch sites and the road conditions in the operation area are known. The roads are represented by the black lines, which can only be driven in one direction, as shown in Figure 1. The average speed of all launchers is 50 km/ h.

Our task is to give a reasonable distribution scheme and maneuvering scheme for 2 waves, so as to minimize the overall exposure time, and 12 launchers each wave are required to launch 12 missiles.

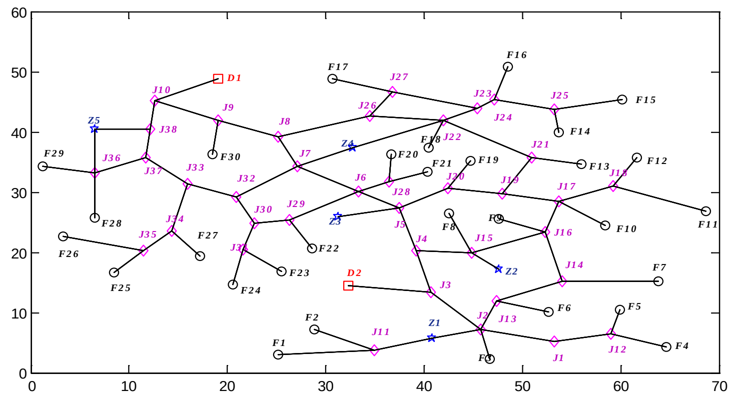

Figure1. the road map of the operate area

2. The Multi-wave optimization model

2.1 Problem analysis

This is an optimization problem of giving a reasonable distribution and maneuvering route. In the first wave, because each launcher from the standby area has been equipped with a missile, it is only necessary to arrange the launcher to launch missile from the standby area to the launch site. After the first wave of mission is completed, the missiles will be loaded in the loading areas. Therefore, for the second wave of mission, it is necessary to arrange all launchers to the nearby loading areas, and then to the nearby launch sites from the loading areas to complete the second wave of mission. Therefore, the "exposure time" in this problem refers to the time from the starting time of each launcher in the standby area to the second wave launching time, as shown in Figure 2.

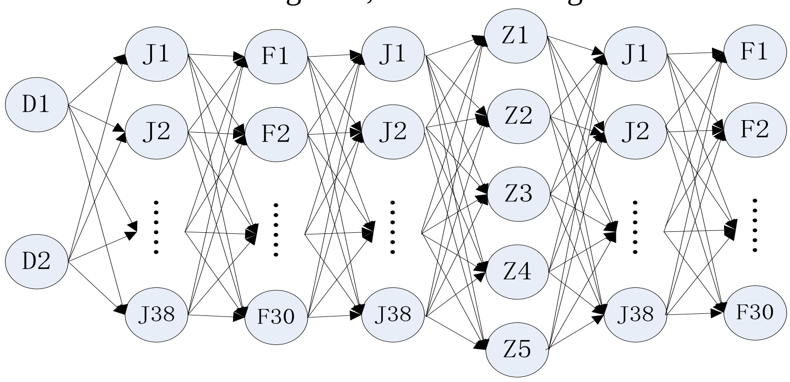

Figure2. the launching missions of 2 waves

2.2The establishment of 0-1 integer programming model

For any laucher, the route is determined by the standby area D_i, the first wave launch site F_{ai}, loading area Z_i and the second wave launch site F_{bi}. So our task is to choose the optimal node combination among all the road node combinations according to the previous idea. That is to say, for all the possible launch sites and the loading areas, an integer programming model of 0-l is established to find the minimum path. A planning model needs to have three elements: decision variables, objective functions and constraints.

(1) Decision Variables

In this question, the decision variable can be expressed as:

$$F_{D_k s,i} = \begin{cases} 1 & \text{Select the i launch site of the s wave from } D_i \\ 0 & \text{Do not select the i launch site of the s wave from the } D_i \end{cases} \tag{1}$$

$$Z_{D_k s,i,j} = \begin{cases} 1 & \text{Select the j loading area of the i launch site of the s wave from } D_i \\ 0 & \text{Do not select the j loading area of the i launch site of the s wave from } D_{\mathrm{i}} \end{cases} \tag{2}$$

(Where, k=1,2; s=1,2; i=1,2…30; j=1,2,…,5)

When all decision variables are integers, this model is called integer programming model. In particular, if the decision variable can only be 0 or 1, the programming model is called 0-1 programming. The model of this problem is a 0-1 programming model.

(2) Objective Functions

The goal of this problem is to minimize the overall exposure time. The exposure time mainly includes three parts: the maneuvering time of the launchers on the road, the node waiting time caused by waiting and the loading area waiting, and the launch waiting time caused by two waves shooting at the same time. In these three parts of exposure time, it is obvious that the road maneuvering time is the main time, and the node waiting time and launch waiting time can be adjusted according to the maneuver scheme, so we all reflect it through the road maneuver time in the model.

When determining the road maneuver time, it is assumed that all launchers are driving at a uniform speed. In this case, the minimum maneuver time is to minimize the sum of the line lengths on all nodes. The objective function can be expressed as:

$$d_{tol} = \sum_{i=1}^{30}\sum_{k=1}^{2} F_{D_k,i} \cdot d_{D_k,i} + \sum_{i=1}^{30}\sum_{j=1}^{5}\sum_{k=1}^{2} F_{D_k 1,i} \cdot Z_{D_k 1,i,j} \cdot d_{FZ,i,j} + \sum_{i=1}^{30}\sum_{j=1}^{5}\sum_{k=1}^{2} F_{D_k 2,i} \cdot Z_{D_k 2,i,j} \cdot d_{FZ,i,j} \quad (3)$$

$d_{D_k,i}$ represents the shortest distance between D_k and the i launch site.

$d_{FZ,i,j}$ represents the shortest distance between the i launch site and the j loading area.

The total maneuvering time can be obtained by dividing the total maneuvering distance by the average speed, and then the total exposure time can be obtained as follow

$$\min T_{tol} = \min(d_{tol} / V) \quad (4)$$

(3) Constraints

There are five constraints in this problem.

On average, 12 launchers are deployed in 2 standby areas, that is, there are exactly 6 launchers per wave in each standby area:

$$\sum_{i=1}^{30} F_{D_k s,i} = 6 \quad (5)$$

The launch site can not be repeated, that is, in the launch site of two waves, each launch point can only be used once:

$$\sum_{k=1}^{2}\sum_{s=1}^{2} F_{D_k s,i} \le 1 \quad (6)$$

The maximum number of missiles in each loading area is 5:

$$\sum_{i=1}^{5} Z_{D_k s,i,j} \le 5 \quad (7)$$

The each loading area corresponds to the launch site:

$$\sum_{j=1}^{5} Z_{D_k s,i,j} = F_{D_k s,i} \quad (8)$$

The each launcher entering the loading area corresponds to the launcher being sent out:

$$\sum_{i=1}^{30} Z_{D_k 1,i,j} = \sum_{i=1}^{30} Z_{D_k 2,i,j} \quad (9)$$

2.2 Problem solving

(1) Combinatorial Optimization of the shortest Route Node

In order to get the optimal node combination, four steps are followed.

Step1:constructing adjacent matrix

The coordinates of all standby areas, road nodes, loading areas, launch sites are extracted. Define variables as follows

$$r_{mn} = \begin{cases} 1 & m \text{ and } n \text{ are connected} \\ +\infty & m \text{ and } n \text{ are not connected} \end{cases} \quad (10)$$

Let $a_1, a_2, \cdots, a_{75}$ correspond to $a_{D1}, a_{D2}, a_{Z1} \cdots a_{Z_5}, a_{J1}, a_{J2}, \cdots, a_{J38}, a_{F1}, a_{F2}, \cdots, a_{F30}$ respectively, taking r_{mn} as an element, the adjacent matrix $A_{(75X75)}$ reflecting the connectivity between two points is obtained.

Step2: Constructing weight matrix

Let $W = (w_{ij})_{75X75}$ be the weighted adjacent matrix, where:

$$w_{ij} = \begin{cases} w(v_i, v_j), & v_i, v_j \in E \\ \infty, & others \end{cases} \quad (11)$$

When the coordinates of each node are known, the weighted adjacent matrix, namely the weight matrix, can be easily obtained through the distance formula between the two points.

Step3: the shortest path of all path nodes

Taking the decision variable x_{ij}, when $x_{ij} = 1$, the arc v_i and v_j are located on the road from vertex 1 to vertex n; Otherwise, it is 0, and its mathematical expression is:

$$\min \sum_{v_i, v_j \in E} w_{ij} \cdot x_{ij}$$

$$s.t. \sum_{j=1}^{n} x_{ij} - \sum_{j=1}^{n} x_{ji} = \begin{cases} 1 & i=1 \\ -1 & i=m \\ 0 & i \neq 1, n \end{cases} \tag{12}$$

There are many ways to find the shortest route. Here, the Dijkstra algorithm is used.

Step4: Solving the 0-1integer programming model

By using Lingo to solve the 0-1 integer programming model, the optional ranges of Da, Fa, Za and Fb corresponding to 12 lines can be directly obtained, and the shortest path can be selected within this range.

(2) The route time planning

The main purpose is to solve the following two problems: the node waiting time caused by waiting and the loading area waiting, and the launch waiting time caused by two waves shooting at the same time.

Step1: the waiting time for launching at the same time

When a launcher is driving on a one-way road, there are two cases: overtaking and meeting. Since all launchers are seen as driving at a uniform speed, it does not take into account the situation of overtaking.

If the launch vehicle is waiting, the waiting time is:

$$t_{wait} = t_{1j} - t_{2j} \tag{13}$$

After the meeting, a waiting time should be added to update the arrival time of the launching vehicle at the node:

$$t_i = d_{FZ,i,j} / v + t_{wait} \tag{14}$$

Step2: the waiting time of the loading areas

As each loading area can only accommodate up to two launchers at the same time, the operating time is 10 minutes. There are two main possible situations, as shown in Figure 3.

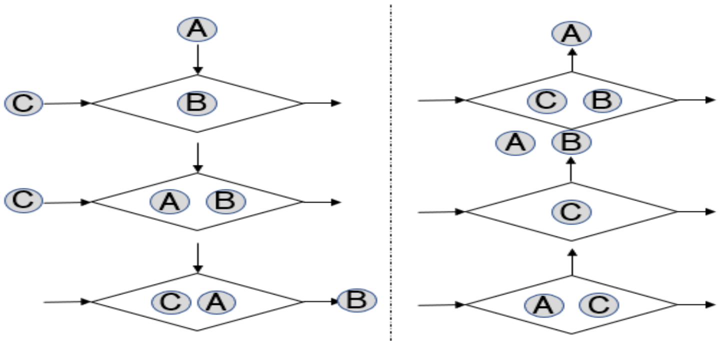

Figure3 the loading waiting diagram

Assuming that the diamond-shaped area is the loading area and the dot is the launcher. The first case is shown on the left side of Figure 3: when there is only one launcher in the loading area, the launcher can continue to stay here until the next one arrives. Another is that if the entry route of the next launcher is the same as the exit route of the launcher in the loading area, the launcher needs to leave the loading area to the road node first, and then the next launcher enters, as shown on the right side of Figure 3. If there is a loading waiting, it is necessary to add a loading waiting time to the exposure time to update the node's departure time:

$$t_i = d_{FZ,i,j} / v + t_Z \tag{15}$$

(3) Waiting time for shooting at the same time of 2 waves

Because the two waves need to shoot at the same time respectively, that is to say, each wave must be based on the longest line time t_{max} of 12 launchers. For the first wave of launch, in order to reduce the exposure time, other launch vehicles can start from the standby area later. The maximum delay time is the longest road time of the first wave minus the time when the launcher reaches the launch point:

$$t_{1\max} - t_j \tag{16}$$

For the second wave, because it involves loading waiting, the launcher can only wait at the final launch site after starting from the loading area, and the waiting time is:

$$t_{2\max} - t_j \tag{17}$$

It is not difficult to find that the waiting of the first wave will not increase the exposure time, while the waiting of the second wave will increase the exposure time.

3. Results

According to the above modeling and solving ideas, we get the missile launch mission planning scheme with two waves. From Table 2, we can see that the 12 launchers with two waves are composed of standby area, launch site, reprint area and the second launch site. The total exposure time of the first wave is 12 hours, 28 minutes and 15 seconds, and the total exposure time of the second wave is 13 hours, 26 minutes and 12 seconds. The shortest exposure time of the two waves was 25 hours, 54 minutes and 27 seconds.

Table2 the Scheme of Optimal Node Combination

launchers	Standby areas	Launch sites I	Loading areas	Launch sites II
1	D2	F17	Z2	F18
2	D1	F16	Z5	F22
3	D1	F28	Z5	F13
4	D2	F26	Z2	F9
5	D2	F25	Z3	F15
6	D1	F24	Z1	F19
7	D2	F12	Z4	F30
8	D2	F10	Z4	F21
9	D1	F1	Z1	F8
10	D2	F6	Z3	F27
11	D1	F7	Z1	F2
12	D1	F4	Z2	F29

4. Conclusion

In the actual mission planning, the missile launch mission planning of two waves can be extended to n waves, in which the connectivity of standby area, launch site, loading area and road nodes can be parameterized. In the process of solving the problem, the mission planning is considered from the perspective of global optimization. But with the increase of waves and the complexity of the problem, it will affect the efficiency of the solution.

According to the recent principle, first of all, the standby area and the loading area are divided by the location, and then the launch point and the reprint area are allocated first, and the local optimal solution is obtained. Then, according to the regional distribution characteristics, the planning scheme is adjusted to get the global optimal solution. In this way, the operation time can be effectively saved and the operation efficiency can be improved.

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